Cauchy Condensation Example

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# Example involving logarithms

Example 1.1

Prove that

diverges.

Solution.

For , define

and for , define

Then, by properties of logarithms,

Also, for , we know that (under the assumption that is an increasing function),

Hence

From lectures (or by applying the Cauchy condensation test again to ), we know that diverges. Hence, by comparison (as ), we find that diverges. Finally, by the Cauchy condensation test, we conclude that

It turns out that this example is a special case of what is known as a *generalised Bertrand series*, and it’s quite surprising how general we can make this example! See [here](https://en.wikipedia.org/wiki/Cauchy_condensation_test#Examples) for details!